## Experiment

## Error Analysis

## INTRODUCTION

In Experiment 1, Motion on an Incline, you may have noticed that the slope of the $v$ - $t$ graph, which we call acceleration, increased as the height of the ramp increased. It seems reasonable that the maximum value of the acceleration could be obtained when the ramp was in a vertical position. In this experiment, you will use different apparatus to determine the acceleration of a freely falling object. Once you have done this, you will address the following questions:

- How do I decide if the value I obtained is "close enough" to the accepted value?
- If I were to repeat the experiment several times, within what range would I expect my values to fall?

This experiment affords you the opportunity to understand variations in experimentally determined data.

## OBJECTIVES

In this experiment, you will

- Determine the value of the acceleration of a freely falling object.
- Compare your value with the accepted value for this quantity.
- Learn how to describe and account for variation in a set of measurements.
- Learn how to describe a range of experimental values.


## MATERIALS

Vernier data-collection interface
Logger Pro or LabQuest App
Vernier Photogate


Figure 1

## PRE-LAB INVESTIGATION

In the previous experiment, you used the time-based data-collection mode in which the software recorded a sensor reading (like position) at even time intervals. In this experiment, you will use a mode in which the software records the elapsed time between some regularly occurring events. When the Picket Fence (a strip of clear plastic with evenly spaced dark bands) passes through a Photogate, the device notes when the infrared beam of the photogate is blocked by a dark band and measures the time elapsed between successive "blocked" states. The software uses these times and the known distance from the leading edge of a dark band to the next to determine the velocity of the picket fence as it falls through the photogate. How does the elapsed time from Blocked state to Blocked state changes as the picket fence accelerates in free fall through the photogate?

## PROCEDURE

1. Connect the photogate to one of the digital inputs on the interface and start the datacollection program. If the photogate has a sliding door, make sure it is open.
2. Check to see if the sensor is working by passing your hand between the infrared LED and the detector. The gate Status should change from "Unblocked" to "Blocked."
3. Fasten the photogate to a support rod or ring stand so that the arms of the photogate are horizontal (see Figure 1).
4. Set up data collection.

Using Logger Pro
a. Choose Data Collection from the Experiment menu.
b. In the Mode list, select Digital Events. End timing after 16 Events.
c. Choose Preferences from the File menu and set the Default Precision for automatic curve fits to four decimal places.

Using LabQuest as a standalone device

- Data collection defaults to Photogate Timing mode with four decimal places when you perform a linear fit.

5. Change the graph setup to view only the velocity $v s$. time graph.
6. Place something soft on the table or floor to cushion the picket fence as it strikes the surface.
7. Hold the picket fence vertically just above the photogate, start collecting data, and release the picket fence. Make sure that it does not strike the photogate as it passes through the arms.
8. Perform a linear fit on the graph of velocity vs. time. Print or sketch a copy of your graph. Take a moment to discuss what the slope and intercept of the line of best fit represent.
9. Based on your discussion, predict whether either of these quantities would change if you were to drop the picket fence through the photogate from a higher point. Test your prediction.
10. To see how repeatable the values of the slope are, repeat Steps 7 and 8 three more times. Be sure to record your values of the slope and intercept in your lab notebook.
11. You may quit the data-collection program for now but do not disassemble your apparatus. You will return to it later.

## EVALUATION OF DATA

1. How do you account for the fact that the values of the slope were nearly the same, whereas the values of the intercept were much more variable?
2. It is highly unlikely that you obtained identical values of the slope of the best-fit line to the velocity $v s$. time graph for each of your trials. How might you best report a single value for the acceleration due to gravity, $a_{g}$, based on your results? Perform the necessary calculation.
3. How does your experimental value compare to the generally accepted value (from a text or other source)? One way to respond to this question is to determine the percent difference between the value you reported and the generally accepted value. Note that if you simplify your units of slope, they will match those of the reported values of $a_{g}$.
4. Your determination of the percent difference does little to answer such questions as, "Is my average value for $a_{g}$ close enough to the accepted value?" or "How do I decide if a given value is too far from the accepted value?" A more thorough understanding of error in measurement is needed. Every time you make a measurement, there is some random error due to limitations in your equipment, variations in your technique, and uncertainty in the best-fit line to your data. Errors in technique or in the calibration of your equipment could also produce systematic error. We'll address this later in the experiment. In order to better understand random error in measurement, you must return to your experimental apparatus to collect more data.
5. Begin the data-collection program as you did before and drop the picket fence through the photogate another 20 times, bringing the total number of trials to 25 . Since you are now investigating the variation in the values of $a_{g}$, you need only record the value of the slope of the best-fit line to the velocity-time graph for each trial.
6. To facilitate the evaluation of the data you have obtained for the 25 trials you have performed, disconnect the photogate from the interface and enter your values of slope into a new Logger Pro file.
7. Determine the average value of $a_{g}$ for all 25 trials. How does this compare with the value you obtained for the first 5 trials? In which average do you have greater confidence? Why?
8. Sort the data in ascending order. Delete the graph of Y vs. slope. Now construct a histogram to display the frequency of the values of $a_{g}$ you have obtained. To do this, choose Additional Graphs Histogram from the Insert menu. In your discussion, you will decide how best to configure the features of the histogram so as to represent the distribution of your values in the most meaningful way.
9. In what range (minimum to maximum) do the middle $2 / 3$ of your values fall? In what range do roughly $90 \%$ of the values closest to your average fall?
10. One way to report the precision of your values is to take half the difference between the minimum and maximum values and use this result as the uncertainty in the measurement. Determine the uncertainty in this way for each range of values you determined in Step 9.
11. In what place (tenths, hundreds, thousandths) does the uncertainty begin to appear? Discuss whether it is reasonable to report values in your average beyond the place in which the uncertainty begins to appear. Round your average value of $a_{g}$ to the appropriate number of digits and report that value plus the uncertainty.

## EXTENSIONS

1. Standard Deviation When you obtained the statistics for your histogram, you may have noticed that, in addition to minimum and maximum values, and the mean and median, the standard deviation (std. dev.) was also provided. Do a web search for standard deviation. From the definition, what fraction of measurements fall within $\pm 1 \sigma$ of the mean; what fraction would fall within $\pm 2 \sigma$ of the mean? Write a brief explanation describing what standard deviation measures. If the standard deviation for your values of slope were four or five times as great as it was, how would that affect the shape of your histogram?
2. Root Mean Square Error When you performed a linear fit on your velocity-time data, you may have noticed that, in addition to the slope and intercept, the software provided the root mean square error (RMSE) for your best fit line. Visit www.vernier.com/til/1014.html to learn what this statistic tells you.
3. Systematic Error Suppose that the clock on the interface ran slow, say at $90 \%$ of its normal rate. How would that change the value you obtained for $a_{g}$ ? You may want to perform another trial and more closely examine the times in your data table to help you answer this question. Would performing more trials fix this problem? Explain.

Consider the case of timing a race with a hand-held stopwatch. How would you expect a hand-held time to compare to one obtained by an electronic timing system? Explain.
4. The value for $a_{g}$ varies according to your location. Do a web search for "Earth gravity" and explain why the value varies.

