

Rotational Dynamics

INTRODUCTION

When you studied Newtonian dynamics you learned that when an object underwent some form of translational motion (whether in a straight line, parabolic, or circular path), the net force applied to the object is proportional to the acceleration. The constant of proportionality is the mass of the accelerating object. When a torque (the rotational analogue to force), is applied to an object that is free to rotate, the object will undergo rotational acceleration. In this experiment, you will investigate the relationship between torque and angular acceleration.

OBJECTIVES

In this experiment, you will

- Collect angular acceleration data for objects subjected to a torque.
- Determine an expression for the torque applied to a rotating system.
- Determine the relationship between torque and angular acceleration.
- Relate the slope of a linearized graph to system parameters.
- Make and test predictions of the effect of changes in system parameters on the constant of proportionality.

MATERIALS

Vernier data-collection interface
Logger *Pro* or LabQuest App
ring stand
lightweight mass hanger
string

Vernier Rotary Motion Sensor
Vernier Rotary Motion Accessory kit
balance
drilled or slotted masses

PRE-LAB INVESTIGATION

1. Connect the sensor to the interface and launch the data-collection program. The default settings work fine for this investigation.
2. Tie a length of string to the edge of the largest pulley on the Rotary Motion Sensor. Pull the string taut and mark the string one meter from the point of attachment. Wind the string on the pulley until the one-meter mark is at the edge of the pulley.
3. Holding the sensor securely, start data collection, then pull the string so that it unwinds from the pulley.
4. Read the maximum angle on the graph of angle *vs.* time and record this value.
5. Repeat Steps 2–4, except that this time, wind the string in the opposite direction.
6. Now tie the string to the edge of the middle pulley on the sensor. Repeat Steps 2–4.

Experiment 13

7. Determine the relationship between the angle through which the pulley turned, the linear distance the edge of the pulley traveled as the string was unwound, and the radius of the pulley. Compare your findings with others in class.

PROCEDURE

1. Connect the Ultra Pulley to the Swivel Mount and then to the Rotary Motion Sensor, as shown in Figure 1. Attach the sensor to a ring stand and position it so that a weight tied to the edge of the large pulley on the sensor and hanging over the Ultra Pulley can hang freely without touching the floor.



Figure 1

Part 1

2. Find the mass of one of the solid aluminum disks, then attach it to the 3-step pulley on the sensor. Record the radius of the pulley.
3. Open a new file in the data-collection program. The default data-collection rate is fine, but reduce the length of the experiment to five seconds.
4. Place the lightest mass available on the hanger, then wind the string onto the largest pulley on the Rotary Motion Sensor.
5. Start data collection, then release the hanging weight. Catch the hanger when the string has completely unwound.
6. To determine the angular acceleration of the disk, perform a linear fit on the appropriate portion of the angular velocity vs. time graph. Record this value in your lab notebook, along with the mass of the hanger and weight for each value you use.
7. Repeat Steps 4–6, increasing the mass of the hanging weight, until you have at least five different values of angular acceleration.

Part 2

8. Find the mass of the second solid aluminum disk. Using the longer machine screw and sleeve, attach both disks to the 3-step pulley on the sensor. Repeat Steps 3–7.

Part 3

9. Remove the disks from the sensor. Find the mass of each of the weights and the rod in the accessory kit. Attach each of the weights to opposite sides of the rod at a distance recommended by your instructor. Record this distance.
10. Attach the rod and weights to the sensor as shown in Figure 2. Repeat Steps 3–7 as you did in Parts 1 and 2.



Figure 2

EVALUATION OF DATA

In order to find a relationship between torque and angular acceleration, you need to know the value of the net torque acting on the system in each of the trials you performed. Since you were not able to measure the torque directly, you must derive an expression you can use to determine the torque from quantities that you *could* measure.

Consider, for a moment, an experiment in which you determined the relationship between net force and the acceleration of an object undergoing translational motion.

$$F_{\text{net}} = ma$$

If you did this experiment with a modified Atwood's apparatus, the force that the earth exerted on a hanging mass accelerated a cart on a track. However, as you may have found, the net force acting on the cart *while it was accelerating*, was less than the weight, mg , of the hanging mass. Consider why this was the case, sketch free body diagrams for both the cart and the hanging weight, then write an expression for the net force acting on the cart as it was accelerating.

From what you have learned about torque, τ , and the relationship between linear and angular acceleration, α , make the appropriate substitutions so you can derive a parallel expression for the net torque acting on the object on the rotating pulley. Your instructor may assist you in this derivation.

Experiment 13

Use of Logger *Pro* will help you to calculate the value of the net torque in Parts 1–3 for your analysis. If you choose to perform the evaluation in LabQuest App, you will need to perform these calculations by hand.

Part 1

1. To evaluate the relationship between acceleration and force, disconnect the sensors from the interface and choose New from the File menu in Logger *Pro*.
2. Enter your values for the hanging mass and angular acceleration. Create a new calculated column finding the values of net torque. Your instructor may guide you in the design of this file.
3. Even though you investigated how angular acceleration responded to changes in the torque, in order to facilitate your analysis, plot a graph of net torque, τ , vs. angular acceleration, α .
4. If the relationship between net torque and angular acceleration appears to be linear, fit a straight line to your data.
5. Write a statement that describes the relationship between the net torque acting on the disk and its angular acceleration.
6. Write the equation that represents the relationship between the net torque, τ , acting on the disk and its angular acceleration, α . Be sure to label this data set with the value of the mass of the disk.

Part 2

7. Choose New Data Set from the Data menu. Enter your values for the hanging mass and angular acceleration for the stacked disks. As you did in Step 2, create a new calculated column finding the values of the net torque the hanging mass applied to the disks.
8. Plot a graph of net torque, τ , vs. angular acceleration, α . If the relationship between net torque and angular acceleration appears to be linear, fit a straight line to your data. If possible, print a copy of the graph showing both data sets.
9. Write the equation that represents the relationship between the net torque, τ , acting on the pair of disks and their angular acceleration, α . Be sure to label this data set with the value of the mass of the disk.
10. How does the slope of this equation compare to the one you obtained in Part 1?

As you are likely to have found before, the slope of a graph is usually some function of physical parameters of the system. For example, in the Newton's Second Law experiment, the slope of the graph of net force vs. acceleration is the *mass* of the object accelerated by the force. The greater the mass of the object, the larger was the force required to produce a given acceleration. In effect, the mass is a measure of the resistance to the change in the motion of the object. Physicists call this resistance to change in motion *inertia*. In this experiment, the slope is also a measure of the resistance of the object to undergo an acceleration; in the case of rotational motion it is known as the *moment of inertia*.

11. From the previous paragraph, you might suspect that the slope is a function of the mass. What evidence do you have that supports this hypothesis?

Part 3

12. Choose New Data Set from the Data menu. Enter your values for the hanging mass and angular acceleration for the rod and weights. As you did in Step 2, determine the values of the net torque the hanging mass applied to the rod and weights.
13. Plot a graph of net torque, τ , vs. angular acceleration, α . If the relationship between net torque and angular acceleration appears to be linear, fit a straight line to your data.
14. Write the equation that represents the relationship between the net torque, τ , acting on the rod and weights and their angular acceleration, α .

Part 4 Further examination of moment of inertia

15. You might also expect that the slope of your graph (moment of inertia) is somehow related to the distance, r , of the object(s) from the axis of rotation. To test this hypothesis, you will need the data from several groups that positioned the weights at different distances along the rod. If that is not feasible, you will have to repeat Part 3 at least five more times, changing the distance the weights are from the center of rotation for each set of net torque vs. angular acceleration.
16. Open a new Logger Pro file and plot moment of inertia, I , vs. radius for the data collected in Part 3. If the relationship appears to be linear, fit a straight line to the data. If not, then take the necessary steps to modify a column so as to produce a linear relationship.
17. How does the slope of *this new* graph compare to the mass of the weights used? Suggest a possible expression for the moment of inertia, I , for the two point masses.
18. Try to account for the fact that your graph has a non-negligible intercept. Your instructor may guide you in making the necessary adjustment to your calculation of I .
19. Now, revisit your equations for Parts 1 and 2. How do the slopes of the lines compare to the expression you obtained for I in Step 17? Suggest a reason for any difference.