

Simple Harmonic Motion

The Mathematical Model for Simple Harmonic Motion

INTRODUCTION

When you suspend an object from a spring, the spring will stretch. If you pull on the object and release it, it will begin to oscillate up and down. In this experiment, you will examine this kind of motion, perform a curve fit on the position-time graph, and relate the parameters of the equation with physical features of the system.

OBJECTIVES

In this experiment, you will

- Collect position vs. time data as a mass, hanging from a spring, is set in an oscillating motion.
- Determine the best fit equation for the position vs. time graph of an object undergoing simple harmonic motion (SHM).
- Define the terms amplitude, offset, phase shift, period and angular frequency in the context of SHM.
- Relate the parameters in the best-fit equation for a position vs. time graph to their physical counterparts in the system.
- Use deductive reasoning to predict the system mass required to produce a given value of angular frequency in the curve fit to SHM.

MATERIALS

Vernier data-collection interface
Logger *Pro* software
Vernier Motion Detector
ring stand and right angle clamp

spring
mass hanger and standard lab masses
wire basket

Experiment 15

PRE-LAB INVESTIGATION

Attach a rod to a vertical support rod using a right-angle clamp. Hang a spring on the horizontal rod, as shown in Figure 1. Now hang a mass hanger from the spring as directed by your instructor. Assume that the bottom of the hanger is the zero position. Pull on the mass hanger slightly and release it. Observe the motion of the hanger. On the axes below, sketch a graph of the position of the hanger as a function of time.

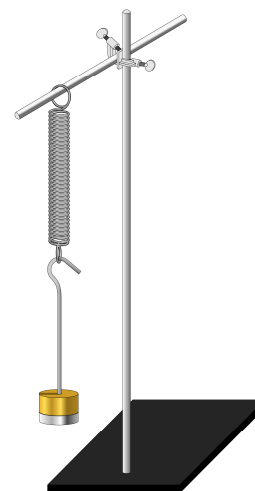
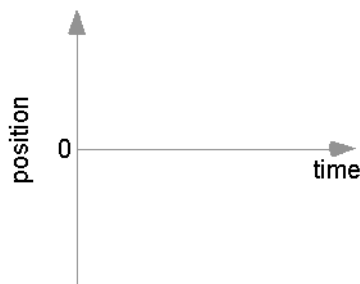


Figure 1

Compare your sketch to those of others in the class.

PART 1 – EXAMINATION OF SIMPLE HARMONIC MOTION

PROCEDURE

1. Connect the Motion Detector to the interface connected to a computer and start *Logger Pro*. Two graphs, position *vs.* time and velocity *vs.* time, will appear in the graph window. For this experiment, you will need only the position *vs.* time graph. Delete the velocity-time graph and choose Auto Arrange from the Page menu.
2. The default data-collection rate is appropriate.
3. If your motion detector has a switch, set it to Track.
4. Place the motion detector on the floor beneath the mass hanger. Place the wire basket over the motion detector to protect it.
5. Lift the hanger and weights slightly, then release. When the hanger is oscillating smoothly, begin collecting data.
6. If the position-time graph appears to be a smooth curve, begin the evaluation of data. If not, repeat until you obtain a smooth curve. If you are satisfied with the data, save this file.



EVALUATION OF DATA

1. Compare the position-time graph you obtained with the one you sketched in the Pre-Lab Investigation. In what ways are the graphs similar? In what ways do they differ? What function appears to describe the position-time behavior of an oscillating body?
2. Before you fit a curve to the position-time graph, turn off Connect Points and turn on Point Protectors. Since the hanger-mass system moves vertically, double-click on the **position** header in the data table and enter **y** as the short name for position.
3. Use the curve-fitting features of *Logger Pro* to fit a sine curve to your data. Write the equation of the representing the motion of the system. Be sure to record the values of the *A*, *B*, *C*, and *D* parameters in the curve fit. Later, you can compare these values to the ones you obtain in Part 2.

PART 2 – EXPLORING THE PARAMETERS OF THE SINE FIT

PROCEDURE

1. Create a new *Logger Pro* file. As before, you will need only the position-time graph. Increase the data-collection rate to 25 Hz. Now, select the Triggering tab and enable Triggering. In your class discussion, determine how to use this feature to control when *Logger Pro* starts data collection; make sure you understand why you have chosen this setting.
2. Hang the mass hanger and the additional masses assigned to you by your instructor from the spring. As before, place the motion detector on the floor beneath the mass hanger and place the wire basket over the motion detector to protect it.
3. Zero the motion detector. Set the masses on the end of the spring in motion, then start data collection. *Logger Pro* will begin collecting data at the appropriate point in the motion of the system. Autoscale the graph. If your position-time graph appears to be a smooth curve, move on to Step 4; otherwise repeat the run.

Exploring Parameter A

4. Turn off Connect Points and turn on Point Protectors, as before. Choose a sine curve fit as you did in Part 1. After you click Try Fit in the Curve Fit dialog box, note the value of *A* suggested by *Logger Pro* to fit the data. Within this dialog box, you can adjust the parameters of the sine fit. Adjustment is made by clicking the + and – buttons to the right of the parameter value. To the right of the + button, you will see a delta (Δ) button. Click this button to control the amount that you change the parameter with the + and – buttons. Change the delta value for the *A* parameter to 0.01. Now click the + and – buttons to see what effect this has on the test plot to fit the data. Cancel to return to the graph window. What aspect of the sine function does the *A* parameter appear to control?
5. Test your conclusion by producing a data set that has an *A* parameter of 0.10 m. Fit a sine curve to the graph and check the *A* parameter. When you have done so, choose Store Latest Run from the Experiment menu and save the experiment file.

Exploring Parameter D

6. Delete the curve fit to your data; perform a curve fit on Run 1. After clicking the Try Fit button, change the value of *D* in the sine fit by increments of ± 0.01 to see what attribute of the sine function is controlled by the *D* parameter.

Experiment 15

7. Test your conclusion by producing a data set that has about the same value of A as before, but with a D parameter of -0.05 m. Choose a sine fit to check D . When you have done so, store this latest run, then choose Hide the Data Set for Run 2. Only your first run should appear on the graph.

Exploring Parameter C

8. Choose a sine function to fit the graph in your first run. As before, click the Try Fit button, then note whether Logger Pro used a value other than 0 for the C parameter. If so, substitute 0 for the value of C and compare this fit to the one originally suggested. If Logger Pro suggested 0 (or a value very close to 0), change C to 6.28 and note the effect on the curve fit.
9. Return the value of C to 0, then increase its value incrementally until you reach 1.6. By what fraction of a complete cycle has the test plot shifted from the original fit to the data? If you are unable to tell for sure, gradually increase C until you reach a value of 3.14 (π). Click Cancel to return to the graph window.
10. To control the value of C in an experiment, you must control the time at which you start collecting data. Choose Data Collection from the Experiment menu and click the Triggering tab. Select the Triggering check box and click On Keyboard to begin data collection.
11. Re-zero the motion detector with the mass at rest, then set the hanger-mass system in simple harmonic motion. Click the Collect button; no data will be collected until you press the space bar. When the hanger has reached the position you believe corresponds to a C value of 1.6, press the space bar. If the value of C in the sine fit does not match your expectations, repeat data collection until you have a run in which C is within ± 0.1 of 1.6. Store this latest run then hide the Data Set for Run 3.

Exploring Parameter B

You may have noticed that none of the changes you made to the system or the time you began collecting data had any effect on the value of B . Taking a closer look at B ; its units must be radians/s in order for the sine function to operate on the argument ($Bt + C$). The B parameter is a measure of how frequently the hanger and mass oscillate.

12. Choose a sine function to fit the graph in your first run. As before, click the Try Fit button, then increase the value of B incrementally to see what effect this change has on the time required to complete one cycle. Now, change B to 6.28, and note the time to complete one cycle. Consider why this is the case.
13. Predict the effect on B of reducing the mass of the hanger and masses to half of its original value. To test your prediction, return the data-collection conditions to those you used when you explored parameters A and D . Re-zero the motion detector, set the lighter hanger-mass system in simple harmonic motion, then collect data.
14. Fit a sine function to your graph as before. Does the value of B agree with your prediction? Store this latest run and save the Logger Pro file; you will return to it later.

EVALUATION OF DATA

To help you evaluate your data, you can return to your *Logger Pro* file and, by selecting More on the vertical axis, you can choose to view the position for any of your saved runs.

Exploring Parameter A

The name given to the A parameter is *amplitude*. What physical aspect of the system did you need to control in order to produce a data set with an amplitude of 0.10 m? Given the function $y = A \sin(Bt + C) + D$, explain why A governs the maximum or minimum value of y in the graph.

Exploring Parameter D

Describe how you modified your experimental apparatus to produce a graph with the D parameter (known as the *offset*) equal to -0.05 m. After having made this change, how could you have produced a graph that was symmetrical about the time axis?

Exploring Parameter C

Considering how you set up *Logger Pro* to collect data, explain why the value of C was very nearly 0 or 2π in your first run. At what point in the cycle did you begin to collect data in order to make $C \sim \pi/2$? At what point in the cycle would you have had to begin collecting data in order to make the value of $C \sim \pi$? Explain.

Exploring Parameter B

The time required to complete a cycle is known as the *period*, T , of oscillation. Divide 2π by T and compare the value to B . The name given to the B parameter is *angular frequency*, ω . From what you know about trigonometry and the unit circle, explain why the period of the sine function was one second when you changed B to 6.28 rad/s.

From a consideration of the elastic and kinetic energy of the oscillating system, it can be shown that $\omega = \sqrt{k/m}$. This explains the fact that reducing the mass to half of its original value did not double the frequency. Calculate what hanging mass would oscillate at double the angular frequency you obtained in your first run. Return to your experimental set up and re-open (if necessary) your *Logger Pro* file. Hide all but your first run, then perform a new run to test your prediction. How close did your results come to your prediction?

Consider any other factors that may have an effect on the value you obtain for B . After your discussion, make the necessary adjustment to the hanging mass to test your prediction and perform another run. Save your *Logger Pro* file in case your instructor wishes to examine it.

EXTENSION – DAMPED HARMONIC MOTION

In this experiment, you stopped collecting data before frictional losses slowed the hanging mass appreciably. Set up another experiment in which you collect data for 10 seconds. Tape half of an index card to the bottom of the mass to provide greater air resistance. Collect data for a run and fit a sine function to the graph as you have done before. In what way does this function no longer match the position-time behavior of the oscillating body?

In a new data set use the Examine tool to record the time and position of the amplitude for each of the peaks in your graph. Fit an exponential function to the max-position *vs.* time data; record the C parameter of the exponential term “ $\exp(-Ct)$ ”. Note that this C is different from the C in the sine fit. Now, return to your first data set, choose Curve Fit and select the sine function. After you choose Try Fit (so that *Logger Pro* suggests values for the parameters) select Define Function. Multiply the existing function by “ $\exp(C*t)$ ” using the value of C you recorded earlier; give the function a sensible name (such as sine-decay). Wait until *Logger Pro* has built this function, then click Try Fit. Wait again until you see a trace appear on the test graph. You may need to replace the values of B and C with the values from your sine fit to the graph. Discuss why this more complex function better describes the position-time behavior of an object undergoing damped harmonic motion.