LabQuest Experiment
15

Simple Harmonic Motion

The Mathematical Model of Simple Harmonic Motion

INTRODUCTION

When you suspend an object from a spring, the spring will stretch. If you pull on the object and release it, it will begin to oscillate up and down. In this experiment, you will examine this kind of motion, perform a curve fit on the position-time graph, and relate the parameters of the equation to physical features of the system.

OBJECTIVES

In this experiment, you will

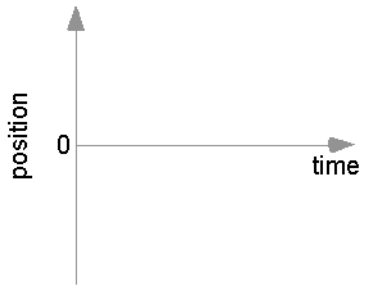
- Collect position *vs*. time data as a mass, hanging from a spring, is set in an oscillating motion.
- Determine the best fit equation for the position *vs.* time graph of an object undergoing simple harmonic motion (SHM).
- Define the terms amplitude, offset, phase shift, period and angular frequency in the context of SHM.
- Relate the parameters in the best-fit equation for a position *vs.* time graph to their physical counterparts in the system.
- Use deductive reasoning to predict the system mass required to produce a given value of angular frequency in the curve fit to SHM.

MATERIALS

Vernier LabQuest
LabQuest App
Vernier Motion Detector
ring stand and right angle clamp
spring
mass hanger and standard lab masses
wire basket

PRE-LAB INVESTIGATION

Attach a rod to a vertical support rod using a right-angle clamp. Hang a spring on the horizontal rod, as shown in Figure 1. Now hang a mass hanger from the spring as directed by your instructor. Assume that the bottom of the hanger is the zero position. Pull on the mass hanger slightly and release it. Observe the motion of the hanger. On the axes below, sketch a graph of the position of the hanger as a function of time.



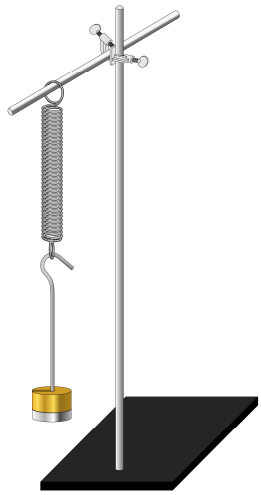


Figure 1

Compare your sketch to those of others in the class.

PROCEDURE

1. Connect the Motion Detector to LabQuest and start a new file in LabQuest App. On the Meter screen, tap Rate and increase the data-collection rate to 50 Hz. Enable Triggering. In your class discussion, determine how to use this feature to control when Logger *Pro* starts data collection; make sure you understand why you have chosen this setting.

2. Hang the mass hanger and the additional masses assigned to you by your instructor from the spring. Place the motion detector on the floor beneath the mass hanger and place the wire basket over the motion detector to protect it.

3. Zero the motion detector. Set the masses on the end of the spring in motion, then start data collection. LabQuest App will begin collecting data at the appropriate point in the motion of the system. If your position-time graph appears to be a smooth curve, store your run and move on to the Evaluation of Data; otherwise repeat the run.

EVALUATION OF DATA

Compare the position-time graph you obtained to the one you predicted in the Pre-Lab Investigation. In what ways are the graphs similar? In what ways do they differ? What function appears to describe the position-time behavior of an oscillating body?

Modeling a curve fit to the data

On the Graph tab choose to view only the position-time graph for this experiment. Since your position-time data appears sinusoidal, you will model a sine function $y = A\sin(Bt + C) + D$ to the data by choosing values of the parameters that will enable you to *manually* fit a curve to your position-time data. The following steps will help you determine the appropriate values of these parameters. You are likely to find that your first effort will produce a model equation that approximates your position-time data. Repeating this process will enable you to refine your

equation to more closely match the data. Before you begin, tap on the position-time graph to determine the maximum and minimum values of position during your run.

Exploring Parameter D

1. Consider the range of values that the sine function returns when it operates on the argument (Bt + C). Under Graph options, choose 1.0 and –1.0 for the top and bottom values displayed on your graph to ensure that your data will appear in the graph window.

2. Choose Model from the Analyze menu, then select $y = A\sin(Bx + C) + D$ as the Equation; keep in mind that the x variable stands for time. The test plot displayed is a much larger version of your sinusoidal y-t data. Using the down arrow, reduce the value of D until the test plot is symmetrical about the time axis. *Do not* tap OK yet. What aspect of the sine fit does the D parameter appear to control? Consider that prior to data collection, you zeroed the motion detector.

Exploring Parameter A

3. While the test plot is now symmetrically placed on the time axis, it is much larger than the plot of your data. Try reducing the value of A until the maximum and minimum values of the test plot approximate those in the graph of your data. What aspect of the sine fit does the A parameter appear to control?

Exploring Parameter C

4. Note that at time $t = 0$, the y-value of your test plot is not 0. Apparently, the default value of 1.0 is not a good match to your data. Try reducing the value of C until the initial value of y equals 0; this also has the effect of making the position of the maximum value of the test plot more closely match one of the maxima in your data. Note this value of C. Now, try increasing C until the test plot is again increasing through 0 when $t = 0$.

5. Since the argument of a sine function must be an angle, the expression $Bt + C$ must have units of an angle measure. The values LabQuest App uses for the C parameter are given in radians. Considering what you know about radian measure and the unit circle, explain why a test plot with a value of $C = 0$ or 2π looks very much like a standard sine curve.

6. Considering how you set up LabQuest App to collect data, explain why it appears that the C parameter for your graph should be ~ 0.

Exploring Parameter B

7. Gradually increase the value of the B parameter. Note the effect this change has on the time required for the object to move through one complete cycle. As B approaches the value that best models your data, you may find that you need to make minor adjustments to C as well. When you think your test plot is a good approximation to the position-time data, tap OK. Record the value of the parameters of the sine fit to your data.

8. Choose Autoscale Once from the Graph menu. Use the Delta function from the Analyze menu and drag across the graph to determine the length of time required for the motion to go through one complete cycle. Do this for a couple of successive cycles and average the value of Δt; then turn off the Delta function. The time required to complete a cycle is known as the *period, T*, of oscillation.

9. Let's take a closer look at *B*; its units must be radians/second in order for the sine function to operate on the argument ($Bt + C$). The *B* parameter, known as *angular frequency,* ω, is a measure of how frequently the hanger and mass oscillate. Given that one cycle is 2π radians, divide 2π by *T* and compare this value to *B*.

10. On the zoomed-in view of the graph, model a sine function that more closely approximates your position-time data. Start with your original parameters, then make slight adjustments to *A, B* and *C*. Use what you learned in Step 9 to choose the value of B. Record the value of your improved parameters.

Revisiting the parameters

11. Return to your apparatus and perform a second run, this time with a smaller amplitude. Store this run. Model a sine fit to these data as you did before and compare the parameters for this run to those from your first run. Which one(s) were different? Explain.

12. Now, reduce the mass hanging from the spring to half of its original value. Perform a third run in which you attempt to keep the amplitude nearly the same as in your second run. When you obtain a run that meets this criterion, store it. Model a sine fit to these data as you did before and compare the parameters for this run to those from your second run. What effect did reducing the mass have on the value of *B*?

13. Save your file before you move on to the Extension.

EXTENSION – PHYSICAL FACTORS AFFECTING *B*

From a consideration of the elastic and kinetic energy of the oscillating system, it can be shown that $\omega = \sqrt{k/m}$. This explains the fact that reducing the mass to half of its original value did not double the frequency. Calculate what hanging mass would oscillate at double the angular frequency you obtained in your first run. Return to your experimental set up and re-open (if necessary) your LabQuest App file. Perform a new run to test your prediction. How close did your results come to your prediction?

Consider any other factors that may have an effect on the value you obtain for *B*. After your discussion, make the necessary adjustment to the hanging mass to test your prediction and perform another run.