Computer Experiment

16

Simple Harmonic Motion

Kinematics and Dynamics of Simple Harmonic Motion

INTRODUCTION

When you suspend an object from a spring, the spring will stretch. If you pull on the object, stretching the spring some more, and release it, the spring will provide a restoring force that will cause the object to oscillate in what is known as simple harmonic motion (SHM). In this experiment, you will examine this kind of motion from both kinematic and dynamic perspectives.

OBJECTIVES

In this experiment, you will

- Collect position *vs.* time data as a weight, hanging from a spring, is set in simple harmonic motion (SHM).
- Determine the best-fit equation for the position *vs.* time graph of an object undergoing SHM.
- Define the terms amplitude, offset, phase shift, period and angular frequency in the context of SHM.
- Predict characteristics of the corresponding velocity *vs.* time and acceleration *vs.* time graphs, produce these graphs and determine best-fit equations for them.
- Relate the net force and acceleration for a system undergoing SHM.

MATERIALS

Vernier data-collection interface
Logger *Pro*
Vernier Motion Detector
Vernier Dual-Range Force Sensor **or**
Wireless Dynamic Sensor System
ring stand and right angle clamp
spring
mass hanger and standard lab masses
wire basket

PRE-LAB INVESTIGATION

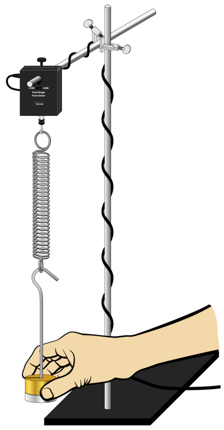
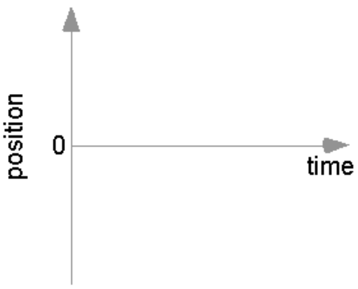


Figure 1

Attach a rod to a vertical support rod using a right angle clamp. Mount a Dual-Range Force Sensor (or WDSS) to the horizontal rod. Now hang a spring from the hook on the sensor and suspend a mass hanger and weights from the spring, as shown in Figure 1. Assume that the bottom of the hanger is the zero position. Pull on the mass hanger slightly and release it. Observe the motion of the hanger. On the axes below, sketch a graph of the position of the hanger as a function of time.



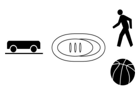
Compare your sketch to those of others in the class.

PROCEDURE

1. Connect the Motion Detector and the Dual-Range Force Sensor to the interface connected to a computer and start Logger *Pro*. Three graphs, force *vs.* time, position *vs.* time and velocity *vs.* time, will appear in the graph window. For now, delete all but the position-time graph. You will be able to insert the others later, when needed. Choose Auto Arrange from the Page menu to re-size the graph.

2. The default data-collection rate is appropriate; however, shorten the duration to 5 seconds.

3. If your motion detector has a switch, set it to Track.



4. Hang the mass hanger and masses from the spring.
   Place the motion detector on the floor beneath the mass hanger. Place the wire basket over the motion detector to protect it.

5. Make sure the hanger is motionless, then zero both the force sensor and the motion detector.

6. Lift the hanger and weights a few centimeters, then release. When the mass hanger is oscillating smoothly, start data collection.

7. If the position-time graph appears to be a smooth curve, autoscale the graph and choose Store Latest Run from the Experiment menu. If not, repeat until you obtain a smooth curve.

8. Perform a second trial, except this time, make the initial displacement of the mass hanger different from what you did for your first trial. If the position-time graph does not appear to be a smooth curve, repeat until you obtain one.

EVALUATION OF DATA

Part 1 Exploration of SHM

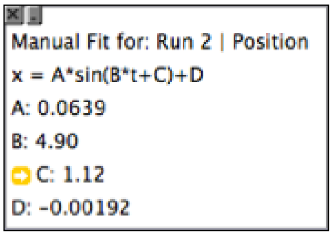
1. Compare the position-time graphs you obtained with the one you sketched in the Pre-Lab Investigation. In what ways are the graphs similar? In what ways do they differ? What function appears to describe the position-time behavior of an oscillating body?

2. Before you fit curves to your position-time graphs, turn off Connect Points and turn on Point Protectors. Since the hanger-mass system moves vertically, double-click on the **position** header in the data table and use ***y*** as the short name for position.

3. Use the curve-fitting features of Logger *Pro* to fit a sine curve to the data for each of your runs. Write the equations that represent the motion of the system for each trial. Be sure to record the values of the *A, B, C* and *D* parameters in the curve fit.

4. Compare the values of the *A* parameter for each of your sine fits. What aspect of the *y-t* graph does *A* appear to describe? The name given to the *A* parameter is *amplitude*.

5. Unless you managed to begin collecting data at the instant the oscillating mass hanger was rising through the 0 position, your *y-t* graphs are likely to be shifted somewhat from the standard position of a graph of $y = \sin\theta$. Delete your curve fits for the *y-t* graphs and choose Curve Fit again for one of your runs. After you click Try Fit in the Curve Fit dialog box, switch to Manual as the Fit Type, then click OK. Doing so enables you to manipulate the parameters in the graph window. When you click on the value of *C* suggested by Logger *Pro* to fit the data, an arrow appears next to *C* as shown to the right. The up and down arrows on your keyboard allow you to change this value. Double-clicking on this window allows you to specify a value for the parameter or the amount of increment. Try increasing and decreasing the value of *C* by ± 0.1 to see what effect this has on the test plot used to fit your data. Return *C* to its original value.



6. Since the argument of a sine function must be an angle, the expression *B*t + *C* must have units of an angle measure. The values Logger *Pro* uses for the *C* parameter are given in radians. What must be the units of *B*? Try changing the value of the suggested *B* parameter to see what effect this has on the number of cycles that appear in the test graph window. The *B* parameter is known as *angular frequency, ω*. Compare the values of *B* for your curve fits to the *y-t* data for both of your runs. Discuss the physical significance of this parameter before moving on to Part 2.

Part 2 Rates of change

In earlier experiments you investigated the relationship between position-time and velocity-time graphs for linear kinematics. In this part you will continue this investigation for the more complex motion of an oscillating body.

7. Hide the data set for Run 2. Choose Insert Graph and then do an Auto Arrange under the Page menu. Select More on the vertical axis of this graph, choose Velocity for the first run, then Autoscale this graph.

8. Select both graphs and choose Group Graphs (x-axes) to make sure that the time axes are aligned. Note the position of the mass hanger when its speed is at a maximum value and again when its speed is zero.

9. Click on the *y-t* graph to make it active and turn on the Tangent Tool. On the *v-t* graph turn on the Examine Tool. Move the cursor across the *y-t* graph; as you do so, compare the slope of the tangent to any point on the *y-t* graph to the value of the velocity on the *v-t* graph. Write a statement describing the relationship between these quantities. When you are finished, de-select these tools.

10. An object's velocity is the rate of change of its position with respect to time. Logger *Pro* does not *measure* the velocity of an object; rather it *calculates* it from the position-time data. Double-click on the column header for velocity to see the equation Logger *Pro* uses to determine velocity. Make sure you understand the function and its argument before you move on.

11. Since cosine is the derivative of the sine (that is, $\frac{d(\sin\theta)}{d\theta} = \cos\theta$), and velocity is the derivative of position, it seems reasonable to use the cosine to fit the velocity-time data. Select the *v-t* graph, choose Curve Fit, then select the sine function as before; this time, however, choose Define Function. In the User Defined Function window replace "sin" with "cos" and name the function "Cosine". After you choose Try Fit, check to see how closely the graph of this function matches that of your data. You may have to try a fit several times before you obtain a nice match. Before you click OK, replace the values of *B* and *C* suggested by Logger *Pro* with those used in the sine fit to your *y-t* graph; make sure the value of *A* is positive.

12. From what you know about the chain rule, determine the value of the coefficient of the cosine function when you take the derivative of your sine fit to the *y-t* graph. Compare this to the *A* parameter suggested by Logger *Pro* for the fit to the *v-t* graph.

13. Insert a new graph and use the Auto Arrange and Group Graphs features as you did in Steps 7 and 8. Choose acceleration for your first run as the vertical axis label and Autoscale the graph. With all three graphs selected, use the Examine tool to note how the position, velocity and acceleration of the hanger change at various times in a given cycle. For example, when the hanger is at its maximum height, what are the values of the velocity and acceleration? When you are done, turn off the Examine tool.

14. From what you know about velocity and acceleration, fit an appropriate function to the acceleration-time graph. In order to keep the keep the argument of the function the same as in the *y-t* and *v-t* graphs, what change do you have to make to parameter *A*? How does *A* compare to the value of the coefficient you obtain when you find the derivative of the function used to fit the *v-t* graph?

Part 3 The role of force

15. On your *v-t* graph replace velocity with force as your vertical axis label. Since you zeroed the force sensor before you began collecting data, this column in the data table ought to be labeled *net* force. Note how the net force acting on the mass hanger varies as its position changes from maximum to minimum. Explain why the net force responsible for SHM is called a *restoring force*.

16. Describe how the acceleration of the mass hanger varies as the net force varies through each cycle of SHM. Would you expect Newton's second law to apply to this type of motion?

17. To test whether the net force is proportional to the acceleration in this kind of motion, change the horizontal axis of the force-time graph to acceleration, then autoscale the graph. Perform a linear fit to the data. Relate the slope to any system parameter that was held constant.

18. Consider the components of the oscillating system when you try to explain any discrepancy between the value of the slope reported by Logger *Pro* and the constant of proportionality you may have expected.

EXTENSION

You should have noticed that the *B* parameter to the sine fit to your *y-t* data for each of your runs was the same. In this activity you will explore aspects of the physical system on which the angular frequency ω, depends.

As you saw in Part 3, Hooke's law describes the relationship between the restoring force and the position of the mass hanger, *y*. Substitution of this expression for force into Newton's second law yields $-k\,y = m\,a$. As you saw in Part 2, the acceleration is the second derivative of position with respect to time. The previous equation can be written as:

$$-k\,y = m\frac{d^2y}{dt^2}$$

This is a 2nd order differential equation. The solution to such an equation is a function. In this experiment, you have found a function for *y(t)* that neatly describes the motion of the system. Substitution of this function in the equation above, rearranging and canceling like terms should enable you to derive an equation for ω in terms of *k* and *m*.

When you have done so, predict how you could change the angular frequency of the SHM by some simple factor (like doubling or halving). Go back to your experimental setup and test your prediction.

Consider any other factors that may have an effect on the value you obtain for *B*. After your discussion, make the necessary adjustment to the hanging mass to test your prediction and perform another run.