LabQuest Experiment
16

Simple Harmonic Motion

Kinematics and Dynamics of Simple Harmonic Motion

INTRODUCTION

When you suspend an object from a spring, the spring will stretch. If you pull on the object, stretching the spring some more, and release it, the spring will provide a restoring force that will cause the object to oscillate in what is known as simple harmonic motion (SHM). In this experiment, you will examine this kind of motion from both kinematic and dynamic perspectives.

OBJECTIVES

In this experiment, you will

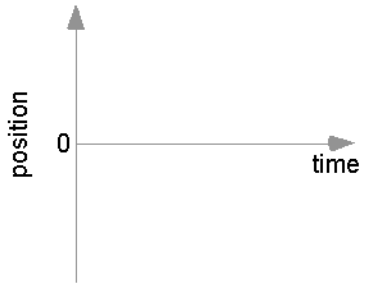
- Collect position *vs.* time data as a weight, hanging from a spring, is set in simple harmonic motion (SHM).
- Determine the best-fit equation for the position *vs.* time graph of an object undergoing SHM.
- Define the terms amplitude, offset, phase shift, period and angular frequency in the context of SHM.
- Predict characteristics of the corresponding velocity *vs.* time and acceleration *vs.* time graphs, produce these graphs and determine best-fit equations for them.
- Relate the net force and acceleration for a system undergoing SHM.

MATERIALS

Vernier LabQuest	ring stand and right angle clamp
LabQuest App	spring
Vernier Motion Detector	mass hanger and standard lab masses
Vernier Dual-Range Force Sensor	wire basket

PRE-LAB INVESTIGATION

Attach a rod to a vertical support rod using a right angle clamp. Mount a Dual-Range force sensor to the horizontal rod. Now hang a spring from the hook on the sensor and suspend a mass hanger and weights from the spring, as shown in Figure 1. Assume that the bottom of the hanger is the zero position. Pull on the mass hanger slightly and release it. Observe the motion of the hanger. On the axes below, sketch a graph of the position of the hanger as a function of time.



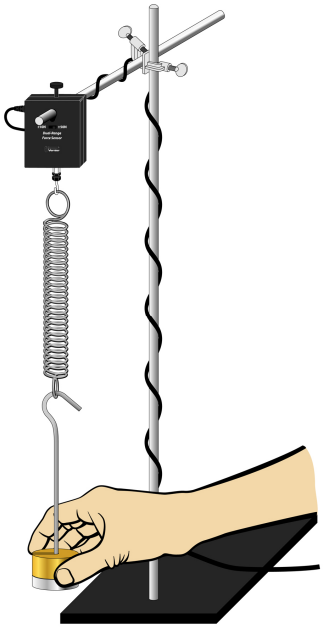


Figure 1

Compare your sketch to those of others in the class.

PROCEDURE

1. Connect the Motion Detector and the Dual Range Force Sensor to LabQuest and start a new file in LabQuest App. The default data-collection rate is appropriate; however, shorten the duration to 5 seconds.
2. If your motion detector has a switch, set it to Track. 
3. Place the motion detector on the floor beneath the mass hanger. Place the wire basket over the motion detector to protect it
4. Make sure the hanger is motionless, then zero both the force sensor and the motion detector.
5. Lift the hanger and weights about ten centimeters, then release. When the mass hanger is oscillating smoothly, begin collecting data.
6. If the position-time graph appears to be a smooth curve, store this run. If not, repeat until you obtain a smooth curve.
7. Perform a second trial, except this time, make the initial displacement of the mass hanger smaller than what you did for your first trial. If the position-time graph is not smooth, repeat until you obtain a smooth curve.

EVALUATION OF DATA

Part 1 Exploration of SHM

On the Graph tab choose to view only the position-time graph for your first run. Since your position-time data appears sinusoidal, you will model a sine function $y = A\sin(Bt + C) + D$ to the data by choosing values of the parameters that will enable you to *manually* fit a curve to your position-time data. The following steps will help you determine the appropriate values of these parameters. You are likely to find that your first effort will produce a model equation that approximates your position-time data. Repeating this process will enable you to refine your equation to more closely match the data. Before you begin, tap on the position-time graph to determine the maximum and minimum values of position during your run.

1. Compare the position-time graphs you obtained with the one you sketched in the Pre-Lab Investigation. In what ways are the graphs similar? In what ways do they differ?

2. Choose Model from the Analyze menu, then select $y = A\sin(Bx + C) + D$ from the drop-down Equation list; keep in mind that the *x* variable stands for time. The test plot displayed is a much larger version of your sinusoidal *y-t* data. In the following steps you will adjust the values of the parameters. *Do not* tap OK until directed to do so.

3. Because you zeroed the motion detector before you began collecting data, your position-time data should be centered about the time axis. Use the down arrow to reduce the value of *D* to 0. This should make the test plot symmetrical with respect to the time axis.

4. Next, reduce the *A* parameter to the value you observed for the maximum value of your position-time graph. What aspect of the *y-t* graph does *A* appear to describe? Tap Cancel, then test your conclusion by performing Steps 2–4 on your second run. Tap Cancel, return to your first run, and adjust *D* and *A* as before. *Do not* yet tap OK. The name given to the *A* parameter is *amplitude*.

5. Unless you managed to begin collecting data at the instant the oscillating mass hanger was rising through the 0 position, your *position-time* graph is likely to be shifted somewhat left or right from the standard position of a graph of $y = \sin\theta$. Try changing the value of the *C* parameter to see what effect this has on the position of the sine curve in the test graph window. Adjust *C* until the test plot appears to have the same initial *y*-value as your data.

6. Since the argument of a sine function must be an angle, the expression $Bt + C$ must have units of an angle measure. The values LabQuest App uses for the *C* parameter are given in radians. Now, take a closer look at *B*; its units must be radians/second in order for the sine function to operate on the argument ($Bt + C$). The *B* parameter, known as *angular frequency,* ω, is a measure of how frequently the hanger and mass oscillate.

7. Gradually increase the value of *B* and note what happens to the time required to complete one cycle– the *period* of oscillation. Continue increasing *B* until the test plot best matches your *y-t* data. You may also need to fine tune your choice for *C*. Tap OK. Be sure to record the values of the parameters to the sine fit to your position-time data.

8. Use the Delta function from the Analyze menu and drag across the graph from one maximum to the next to determine the *period, T*, of oscillation. Given that one cycle is 2π radians, divide 2π by *T* and compare this value to *B*. Express angular frequency, ω, in terms of *T*. Make sure that you discuss the physical significance of this parameter before moving on to Part 2 of the evaluation. At this point you may turn off the Model function for the position-time graph.

Part 2 Rates of change

In earlier experiments you investigated the relationship between position-time and velocity-time graphs for linear kinematics. In this part you will continue this investigation for the more complex motion of an oscillating body.

9. From the Graph menu, choose to show both graphs. Change the vertical axis of Graph 1 to velocity. Note the position of the mass hanger when its speed is at a maximum value and again when its speed is zero.

10. Turn on Tangent (found in the Analyze menu) and tap the position-time (*y-t*) graph at several places. Compare the slope of the tangent to any point on the *y-t* graph to the value of the velocity on the *v-t* graph. Write a statement describing the relationship between these quantities, then turn off Tangent.

11. The instantaneous rate of change of a function is known as its derivative. Since cosine is the derivative of the sine (that is, $\frac{d(\sin\theta)}{d\theta} = \cos\theta$), and velocity is the derivative of position, it seems reasonable to use the cosine to fit the velocity-time data. Choose to Model the *v-t* graph, then select the cosine function. Adjust the *D* parameter to 0 as before and reduce the *A* parameter to match the amplitude. Then, adjust the *B* and *C* parameters to the values used in the sine fit to the *y-t* data; this makes the arguments of the two functions the same.

12. Tap OK. From what you know about the chain rule, determine the value of the coefficient of the cosine function when you take the derivative of your sine fit to the *y-t* graph. Compare this to the *A* parameter you used to fit the cosine to the *v-t* graph.

13. Now change the vertical axis label of the position-time graph to acceleration and choose Autoscale Once to make sure that you can see the data points. Examine the *v-t* and *a-t* graphs to note how the velocity and acceleration of the hanger change at various times in a given cycle.

14. From what you know about velocity and acceleration, use the Model function to fit an appropriate function to the acceleration-time graph. Make sure that you keep the argument of the function the same as in the *y-t* and *v-t* graphs. What change do you have to make to parameter *A* in order to match the test plot to your *a-t* graph? How does *A* compare to the value of the coefficient you obtain when you find the derivative of the function used to fit the *v-t* graph? Tap OK to return to the Graph tab.

Part 3 The role of force

15. On your *v-t* graph replace velocity with force as your vertical axis label. Change the vertical axis label of the *a-t* graph to position. Since you zeroed the force sensor before you began collecting data, this column in the data table ought to be labeled *net* force. Note how the net force acting on the mass hanger varies as its position changes from maximum to minimum. Explain why the net force responsible for SHM is called a *restoring force*.

16. Change the vertical axis of the position-time graph back to acceleration (you may need to Autoscale to view the data points). Describe how the acceleration of the mass hanger varies as the net force varies through each cycle of SHM. Would you expect Newton's second law to apply to this type of motion?

17. To test whether the net force is proportional to the acceleration in this kind of motion, choose to view just the force-time graph. Tap the y-axis label and select Acceleration. Turn off Connect Points in the Graph Options dialog box. From the Analyze menu, choose Curve Fit and perform a linear fit to the data. Relate the slope to any system parameter that was held constant.

18. Consider the components of the oscillating system when you try to explain any discrepancy between the value of the slope reported by LabQuest App and the constant of proportionality you may have expected.

EXTENSION

You should have noticed that the *B* parameter to the sine fit to your *y-t* data for each of your runs was the same. In this activity you will explore aspects of the physical system on which the angular frequency ω, depends.

As you saw in Part 3, Hooke's law describes the relationship between the restoring force and the position of the mass hanger, *y*. Substitution of this expression for force into Newton's second law yields $-k\,y = m\,a$. As you saw in Part 2, the acceleration is the second derivative of position with respect to time. The previous equation can be written as:

$$-k\,y = m\frac{d^2y}{dt^2}$$

This is a 2nd order differential equation. The solution to such an equation is a function. In this experiment, you have found a function for *y(t)* that neatly describes the motion of the system. Substitution of this function in the equation above, rearranging and canceling like terms should enable you to derive an equation for ω in terms of *k* and *m*.

When you have done so, predict how you could change the angular frequency of the SHM by some simple factor (like doubling or halving). Go back to your experimental setup and test your prediction.

Consider any other factors that may have an effect on the value you obtain for *B*. After your discussion, make the necessary adjustment to the hanging mass to test your prediction and perform another run.