Experiment
17

Pendulum Periods

INTRODUCTION

The introductory treatment of the motion of a pendulum leaves one with the impression that the period of oscillation is independent of the mass and the amplitude, and depends only on the length of the pendulum. These relationships are generally true so long as two important conditions are met:

1. the amplitude is small (<< 1 radian), and
2. the mass of the system is concentrated at the end of the string.

In this experiment and the next you will examine the behavior of a pendulum in greater detail to see what occurs when these conditions are no longer true. You will examine the approximations made to simplify the analysis of the pendulum and determine when and why these approximations begin to break down. The first of these is the subject of this experiment; the second will be examined in Experiment 18.

OBJECTIVES

In this experiment, you will

- Collect angle *vs.* time data for a simple pendulum.
- Determine the best-fit equation for the angle *vs.* time graph.
- From an analysis of the forces acting on the pendulum bob, derive the equation describing the motion of the pendulum.
- Relate the parameters in the best-fit equation for the angle *vs.* time graph to their physical counterparts in the system.
- Determine the period of oscillation from an analysis of the angle *vs.* time graph.
- Account for the deviation from constant periods when the amplitude becomes large.

MATERIALS

Vernier data-collection interface	vertical support rod and clamp
Logger *Pro* or LabQuest App	right-angle clamp
Vernier Rotary Motion Sensor	protractor
Vernier Rotary Motion Accessory Kit	metric ruler or tape

PRE-LAB INVESTIGATION

Attach a rod to a vertical support rod using a right-angle clamp. Attach the aluminum rod from the Accessory Kit to the Rotary Motion Sensor, then attach the sensor to the horizontal rod, as shown in Figure 1. Now attach one of the heavy cylindrical weights from the accessory kit to the bottom of the rod as shown. Assume that when the rod is hanging vertically the weight is in the zero position. Pull on the weight slightly and release it. Observe the motion of the weight. On the axes below, sketch a graph of the angular position (in degrees or radians) of the weight as a function of time.

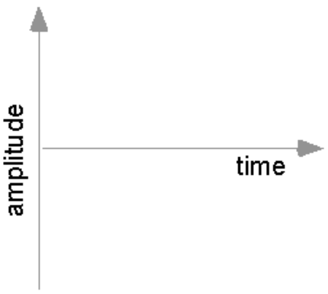


Figure 1

Compare your sketch to those of others in the class.

PART 1 – EXAMINATION OF THE MOTION OF A SIMPLE PENDULUM AT VARIOUS AMPLITUDES

PROCEDURE

1. Make sure that the vertical support rod for the Rotary Motion Sensor is securely attached to a bench or table. When the pendulum is set in motion the sensor should be stationary.

2. Measure the distance between the point of attachment of the rod to the 3-step pulley and the center of mass of the weight at the end of the rod. Record this value as the length, *l,* of the pendulum.

3. Connect the sensor to the interface and start the data-collection program. Two graphs: angle *vs.* time and velocity *vs.* time will appear in the graph window. For this experiment, you will need to view only the angle *vs.* time graph.

4. The default data-collection rate is appropriate. However, you should increase the resolution of the sensor by selecting the X4 mode.

    - In Logger *Pro*, choose Set Up Sensors from the Experiment menu. Once you select your interface, click on the icon for the RMV and then select X4 Mode.
    - In LabQuest App, tap on the meter window and then select the X4 Mode.

5. Because the default data-collection mode automatically resets the zero position when you start data collection, it is unnecessary to manually zero the sensor before collecting data. However, the bob must be motionless before you begin data collection.

6. Start data collection. Then, using a protractor to measure the angle, pull the rod through a 5° angle and release. Be sure that the pendulum swings freely for at least five seconds. Store this run.

7. Repeat Step 6 for amplitudes of 10°, 15° and 20°, storing each run. Save this file; you will return to it later in the experiment.

EVALUATION OF DATA

1. Compare the angle-time graph you obtained with the one you sketched in the Pre-Lab Investigation. In what ways are the graphs similar? In what ways do they differ? How does the angle-time graph compare to what you have seen for simple harmonic motion?

Determination of ω using Logger *Pro*

2. Before you fit a curve to the position-time graph, turn off Connect Points and turn on Point Protectors.

3. Drag-select that portion of the graph for your first run where the bob is swinging freely.Fit a sine curve to these data. Record the value of the *B* parameter to the sine fit. Repeat this process for your other runs.

4. The *B* parameter is the angular frequency, ω, for this oscillation. Does the value of ω appear to depend on the amplitude of the oscillation? Determine an average value of ω for your four runs.

Determination of ω using LabQuest App

2. Choose to view your first run. Choose the Delta function under the Analysis menu. Drag-select 4–5 cycles of the graph where the bob is swinging freely. The time (Δx) divided by the number of cycles gives the period of oscillation, *T*. Record this value. Repeat this process for your other runs.

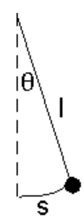
3. The angular frequency, ω, for this oscillation is given by $\omega = \dfrac{2\pi}{T}$. Determine the value of ω for each of your runs. Record these values.

4. Does the value of ω appear to depend on the amplitude of the oscillation? Determine an average value of ω for your four runs.

Determination of the equation of motion for the pendulum.

5. Sketch a force diagram for the bob when it is displaced to one side. Write the expression for the restoring force, the component of the gravitational force that opposes the displacement through angle θ.

6. Write the Newton's second law equation describing the motion of the bob once it is released. Express the acceleration as the second derivative of the arc length, *s,* with respect to time.[1] Given that $s = l\theta$, express the acceleration in terms of θ, *l,* and *t*.



[1] Recall that $v = \dfrac{ds}{dt}$ and that $a = \dfrac{dv}{dt}$.

Keep in mind that the acceleration vector for the pendulum always acts in a direction opposing the displacement through angle θ.

7. Simplify your second equation and rearrange the terms so that you have set the equation equal to 0. You have now produced a 2nd order differential equation describing the motion of the pendulum bob. Check your equation with those of others in your class.

8. In your class discussion make sure that you determine and understand the simplification necessary to suggest a solution to the equation you have derived.

Relating ω to physical features of the system

9. The coefficient of t in your solution to the differential equation is the angular frequency, ω, of the motion. From the known values of g and l, calculate the value of ω you would expect for your pendulum. Compare this to the average value of ω you obtained from the curve fit to your data. Determine the percent difference between these values.

10. Slide the weight a few centimeters higher on the rod. Measure the new length, l, of the pendulum. Calculate the expected value of ω for this pendulum.

11. Collect angle-time data for another run using an amplitude between 10 and 20°. Determine ω as you did before. Compare this value to the one you calculated.

PART 2 – EFFECT OF AMPLITUDE ON PERIOD

PROCEDURE

1. Return the weight to its original position at the end of the aluminum rod. Re-open your experiment file from Part 1.

2. Collect angle *vs.* time data for the pendulum as before with an amplitude of 25°. Determine the period, T, of the oscillation.
   - In Logger *Pro*, perform a sine curve fit on the appropriate part of the angle-time graph. Record the value of ω for this run. Leaving the curve fit information window open speeds up this process for subsequent runs.
   - In LabQuest App, determine T as you did in Part 1.

3. Repeat step 2, increasing the amplitude by 5° each time until you reach 60°. It is unnecessary to store these runs. Beyond this angle, increase the amplitude by 10° until you reach 120°. Record the period for each run.

EVALUATION OF THE DATA

1. Disconnect the sensor from the interface and choose New from the File menu.
   - In Logger *Pro*, manually enter the frequency-amplitude data and use a calculated column to determine the period.
   - In LabQuest App, manually enter the period-amplitude data.

2. Examine the graph of period *vs.* amplitude. Be sure to scale the vertical axis from zero.

3. Examine the statistics on the first four data points for this graph. Perform a linear fit for this same portion of the graph. Note at what point the data show that the period is no longer independent of amplitude. Save this file.

4. Reflect on the simplifying assumption that you made in Part 1 that allowed you to solve the differential equation. Test the validity of your assumption by examining the relationship between θ and $\sin\theta$ for the range of values used in your experiment. Use radian measure for your angles. At what point does your approximation begin to break down?

EXTENSION

Because its period of oscillation is very nearly constant for small amplitudes, the simple pendulum has been used in timekeeping since the time of Huygens. You can test this reliability yourself by collecting angle-time data for a much longer time (say 1 or 2 minutes), then determine the period for short intervals within the duration of your experiment. Try this for angles for which your initial approximation is valid, then for a couple of angles outside the range of validity. What do you find?