Experiment
18

Physical Pendulum

INTRODUCTION

The introductory treatment of the motion of a pendulum leaves one with the impression that the period of oscillation is independent of the mass and the amplitude, and depends only on the length of the pendulum. These relationships are generally true so long as two important conditions are met:

1) the amplitude is small (<< 1 radian), and
2) the mass of the system is concentrated at the end of the string.

Experiment 17 examined the non-ideal behavior of the pendulum when the amplitude was not kept small. In this experiment, you will investigate the effect on the behavior of a pendulum when the mass of the system can no longer be treated as a point mass at the end of a massless string.

OBJECTIVES

In this experiment, you will

- Collect angle *vs.* time data for a variety of physical pendulums.
- Determine the period of oscillation from an analysis of the angle *vs.* time graph.
- From an analysis of the torques acting on the system, derive the equation describing the motion of the physical pendulum.
- Compare this equation to the one that describes the motion of a simple pendulum.
- Relate the angular frequency, ω, of the system to its physical features.
- Compare the agreement between experimental and calculated values of ω determined by this treatment of the system with those obtained by treating the system as if it were a simple pendulum.

MATERIALS

Vernier data-collection interface
Logger *Pro* or LabQuest App
Vernier Rotary Motion Sensor
Vernier Rotary Motion Accessory Kit
vertical support rod and clamp
right-angle clamp
protractor
metric ruler or tape
balance

PRE-LAB INVESTIGATION

For a simple pendulum (a weight at the end of a light string) the analysis of forces acting on the weight yields an equation of motion in which the angular frequency is given by $\omega = \sqrt{g/l}$. **Note:** The mass of the bob does not appear in this expression.

Consider the physical pendulum system shown in Figure 1. The aluminum rod is certainly not a light string. Discuss why an analysis of forces alone is insufficient to determine the angular frequency of this system. What approach do you think would be more fruitful?

Figure 1

PART 1 – A CLOSER LOOK AT A SIMPLE PENDULUM

PROCEDURE

1. Find the mass of the aluminum rod and the cylindrical weight with thumbscrew. [1]

2. Set up the apparatus pictured in Figure 1. Make sure that the vertical support rod for the Rotary Motion Sensor is securely attached to a bench or table. When the pendulum is set in motion the sensor should be stationary.

3. Measure the distance between the point of attachment of the rod to the 3-step pulley and the center of mass of the weight at the end of the rod. Record this value as the radius, *r,* of the rotation of the weight.

4. Connect the sensor to the interface and start the data-collection program. Two graphs: angle *vs.* time and velocity *vs*. time will appear in the graph window. For this experiment, you will need to view only the angle *vs.* time graph.

5. The default data-collection rate is appropriate. However, you should increase the resolution of the sensor by selecting the X4 mode.

    - In Logger *Pro*, choose Set Up Sensors from the Experiment menu. Once you select your interface, click on the icon for the RMV and then select X4 Mode.
    - In LabQuest App, tap on the meter window and then select the X4 Mode.

6. Because the default data-collection mode automatically resets the zero position on Collect, it is unnecessary to manually zero the sensor before collecting data. However, the bob must be motionless before you begin data collection.

[1] If you have performed Experiment 17, open your saved data file from Part 1. Then once you have completed Step 1, you may proceed with the Evaluation of Data.

7. Start data collection. Then, using a protractor to measure the angle, pull the rod through a 5° angle and release. Be sure that the pendulum swings freely for at least five seconds. Store this run. Do not be too concerned if the graph is not a smooth curve; subsequent trials with larger amplitudes will produce smoother graphs.

8. Repeat Step 7 for amplitudes of 10°, 15° and 20°, storing each run. Save this file; you will return to it later in the experiment.

EVALUATION OF DATA[2]

Determination of ω using Logger *Pro*

1. Before you fit a curve to the position-time graph, turn off Connect Points and turn on Point Protectors.

2. Drag-select that portion of the graph for your first run where the bob is swinging freely. Use the curve-fitting features of Logger *Pro* to fit a sine curve to these data. Record the value of the *B* parameter to the sine fit. Repeat this process for your other runs.

3. The *B* parameter is the angular frequency, ω, for this oscillation. Does the value of ω appear to depend on the amplitude of the oscillation? Determine an average value of ω for your four runs.

Determination of ω using LabQuest App

1. Choose to view your first run. Choose the Delta function under the Analysis menu. Drag-select 4–5 cycles of the graph where the bob is swinging freely. The time (Δx) divided by the number of cycles gives the period of oscillation, *T*. Record this value. Repeat this process for your other runs.

2. The angular frequency, ω, for this oscillation is given by $\omega = \frac{2\pi}{T}$. Determine the value of ω for each of your runs. Record these values.

3. Does the value of ω appear to depend on the amplitude of the oscillation? Determine an average value of ω for your four runs.

Determination of the equation of motion for the pendulum.

4. The physical pendulum can be viewed as an extended body that rotates about the point at which the rod is connected to the pulley. For now, neglect the rod and treat the weight as a point mass. Sketch a force diagram for the weight at the end of the rod when it is displaced to one side. Write the expression for the torque that acts on the weight to restore it to its original position when released.

5. Write the Newton's second law equation describing the rotation of the weight about the pivot point once it is released. In other words, relate the torque from the gravitational force on the pendulum to its angular acceleration. Consider the distance to the center of mass and use that as the point of action for the gravitational force. Express the angular acceleration as the

[2] If you have performed Experiment 17, review your determination of ω for your runs with the amplitude ranging from 5° – 20°. Then move on to Step 4.

second derivative of the angle θ, with respect to time.[3] Keep in mind that the acceleration vector for the pendulum always acts in a direction opposing the displacement through angle θ.

6. Divide through by I in your second equation and rearrange the terms so that you have set the equation equal to 0. You have now produced a 2nd order differential equation describing the motion of the pendulum bob. Check your equation with those of others in your class.

7. In your class discussion make sure that you understand the simplification necessary to suggest a solution to the equation you have derived.

8. Using the expression for the moment of inertia, I, of a point mass, further simplify the equation above. Compare this to the equation of motion for a simple pendulum derived from a consideration of linear dynamics only.

9. Now, consider how the inclusion of the torque and moment of inertia of the rod would change the general equation of motion for the physical pendulum.

PART 2 – EFFECT OF POSITION OF THE WEIGHT

Before you move on to this part of the experiment, make sure that you understand how the angular frequency, ω, depends on both the total torque and the total moment of inertia of the *system*. You may need to review the expression for the moment of inertia of a rod.

Both the torque acting on the weight and its moment of inertia depend on its position on the rod. By contrast, both the torque and the moment of the inertia for the aluminum rod are constant. It is recommended that you set up a Logger *Pro* file to facilitate calculation of the torque and the moment of inertia of the system as well as the expected value of the angular frequency, ω. This file will enable you to compare the expected value to your experimentally determined value of ω for a variety of configurations of your physical pendulum. Your instructor may guide you in the design of this file. See the sample column headers below.

radius (m)	I-weight (kgm^2)	I-total (kgm^2)	τ-weight (m-N)	τ-total (m-N)	ω-torq (rad/s)	ω-meas (rad/s)	%-diff	ω-force (rad/s)	%-diff-2

The **%-diff** columns allow you to compare the agreement between the experimentally determined value of ω and those calculated by treating the system using either rotational or linear dynamics.

When you have set up the Logger *Pro* file, you are ready to test your predictions.

PROCEDURE

1. Re-open your experiment file from Part 1.

[3] Recall that $a = \dfrac{d^2\theta}{dt^2}$.

2. Move the weight up the rod. Measure the length between the pivot point and the center of mass of the weight and record this value.

3. Collect angle *vs.* time data for the pendulum as before using an amplitude of 10–15°. Determine the angular frequency as you did in Part 1.

4. Repeat Step 3, decreasing the length gradually until you have data for at least 8 different lengths.

EVALUATION OF THE DATA

1. Re-open the Logger *Pro* file you built earlier.

2. Enter your values for radius and ω-meas that you recorded.

3. Compare the %-differences between measured and calculated values of ω treating the system as a physical pendulum as opposed to a simple one.

PART 3 – EFFECT OF ADDING ANOTHER WEIGHT TO THE SYSTEM

PROCEDURE

1. Re-open your experiment file from Part 2.

2. Add the second cylindrical weight to the rod as shown in Figure 2. Determine the distance between the pivot point and the center of mass of this second weight; label this as radius-2.

3. Collect angle *vs.* time data for the pendulum as before using an amplitude of 10–15°. Determine the angular frequency as you did in Part 1.

4. Repeat Step 3 for several new positions of the two weights.

Figure 2

EVALUATION OF DATA

1. Re-open the Logger *Pro* file you created earlier. Label your first data set: One Mass on Rod.

2. Choose New Data Set from the Data menu. Label this set: Two Masses on Rod. Add a new manual column, radius-2, and two new calculated columns, *I*-weight2 and τ-weight2, with the appropriate equations. Modify the equations used to calculate *I*-total and τ-total**.**

3. Choose Table from the Insert menu. Choose Table Options from the Options menu. De-select your first data set, then select the second set to be displayed in the table.

4. Enter your measured values of radius-1 and radius-2.

5. How do your measured values of ω compare to the values you have calculated?

EXTENSION

In Part 3 of this experiment, one end of the rod was attached to the rotary motion sensor and the weights were placed at various positions near the other end. Consider how the system would behave if you were to attach the center of the rod to the sensor and place weights on either side, as shown in Figure 3. How would this configuration affect the moment of inertia of the system and the torque acting on it? Once you have made your predictions, you might try collecting data with the weights at different positions to see how these configurations affect the angular frequency of the pendulum.

Figure 3